

WHAT IS CLAIMED IS:

1 1. A method for computing distances between a received point and four points in a two-dimensional grid with a constellation representing a number of bits greater than three, wherein
2 each of the four points belong to a unique coset in the constellation, the method comprising:

4 determining a first point on a grid nearest to the received point;
5 computing a second point closest to the received point inside a specified area;
6 computing a third, fourth, and fifth point, wherein each point is a member of a different
7 coset and each point is the closest point in its coset to the received point; and
8 computing a distance from the received point to each of the second, third, fourth, and
9 fifth points.

1 2. The method of claim 1 further comprising after the first computing, recomputing the
2 second point if the second point is invalid.

1 3. The method of claim 2, wherein the second point is invalid if it is outside of the
2 constellation.

1 4. The method of claim 1, wherein the first point can be determined by evaluating:
2 $\text{round}((Rx + iRy - 1 - i)/2^2 + 1 + I)$,
3 wherein Rx and Ry are two-dimensional components of the received point, i is the imaginary
4 number, and round(.) is an operator that returns an integer number closest to a value provided to
5 it.

1 5. The method of claim 1, wherein the number of bits is an even value, wherein the received
2 point can be expressed in two-dimensional components Rx and Ry, and wherein the first

3 computing comprises:

4 determining if Rx and Ry lie inside a square specified by the number of bits; and
5 computing two-dimensional components of the second point based on the second
6 determining.

1 6. The method of claim 5, wherein the second determining comprises:

2 setting Cx = 1 if Rx lies inside a boundary of the square, else Cx = -1;

3 setting Cy = 1 if Ry lies inside a boundary of the square, else Cy = -1;

4 and wherein the fourth computing comprises

5 setting Ax = sign(RGx) * MAX_{XY} if Cx = -1, else Ax = RGx; and

6 setting Ay = sign(RGy) * MAX_{XY} if Cy = -1, else Ay = RGy,

7 wherein Ax and Ay are two-dimensional components of the second point, RGx and RGy are
8 two-dimensional components of the first point, MAX_{XY} is value describing the size of the square
9 and can be computed by $2^{\text{number of bits}/2} - 1$.

1 7. The method of claim 5, wherein the second computing comprises:

2 computing an intermediate value, d, wherein d = the received point – the second point;

3 setting the third point = the second point + Cx * sign(dx) * 2;

4 setting the fourth point = the second point + i * Cy * sign(dy) * 2; and

5 setting the fifth point = the second point + 2(Cx * sign(dx) + i * Cy * sign(dy)),

6 wherein Cx and Cy are values specifying if the two-dimensional components of the received
7 point lie inside a boundary of the square and dx and dy are two-dimensional components of d.

1 8. The method of claim 5, wherein the third computing comprises computing a Euclidean
2 distance from the received point to each of the second, third, fourth, and fifth points.

1 9. The method of claim 8, wherein each of the second, third, fourth, and fifth points belong
2 to a unique coset.

1 10. The method of claim 1, wherein the number of bits is an odd value greater than three,
2 wherein the received point can be expressed in two-dimensional components Rx and Ry, and
3 wherein the first computing comprises computing two-dimensional components of the second
4 point, wherein the second point lies within a first square.

1 11. The method of claim 10, wherein the first square encompasses a cross-shaped
2 constellation, wherein a value MAX_{XY} describes the size of the first square and can be computed
3 by expression $(2^{\lceil \text{number of bits}/2 \rceil} + 2^{\lfloor \text{number of bits}/2 \rfloor})/2 - 1$, wherein a second square that is a largest
4 square encompassing 2^N constellation points with N being an even integer that can be enclosed
5 in the first square, wherein MAX_X and MAX_Y describe the size of the second square, and
6 wherein the first computing comprises:

7 setting Ax = sign(RGx) * MAX_{XY} if abs(RGx) > MAX_{XY} else Ax = RGx; and
8 setting Ay = sign(RGy) * MAX_{XY} if abs(RGy) > MAX_{XY} else Ay = RGy,
9 wherein Ax and Ay are two-dimensional components of the second point, and RGx and RGy are
10 two-dimensional components of the first point.

1 12. The method of claim 10 further comprising after the first computing, recomputing the
2 second point if the second point is invalid.

1 13. The method of claim 12, wherein the second point is invalid if (abs(Rx) > MAX_X and
2 abs(Ry) > MAX_Y) is true.

1 14. The method of claim 12, wherein the recomputing comprises
2 setting the second point = $Ax + i * \text{sign}(Ay) * \text{MAX}_Y$
3 and a sixth point = $\text{sign}(Ax) * \text{MAX}_X + i * Ay$ if $\text{abs}(Rx) > \text{abs}(Ry)$
4 else setting the second point = $\text{sign}(Ax) * \text{MAX}_X + i * Ay$
5 and the sixth point = $Ax + i * \text{sign}(Ay) * \text{MAX}_Y$,
6 wherein Ax and Ay are two-dimensional components of the invalid second point and α is
7 another point in the two-dimensional grid.

1 15. The method of claim 14, wherein the second computing comprises:
2 determining if the second point lies on the boundaries of the first square and α lies on the
3 boundaries of a second square, wherein the second square is the largest square encompassing 2^N
4 constellation points with N being an even integer that can be enclosed in the first square, wherein
5 MAX_X and MAX_Y describe the size of the second square; and
6 computing the third, fourth, fifth points.

1 16. The method of claim 15, wherein the second determining comprises:
2 setting $Cx = -1$ if ((($\text{abs}(Rx) \geq \text{MAX}_X$) and ($\text{abs}(Ay) > \text{MAX}_Y$)) or ($\text{abs}(Rx) \geq$
3 MAX_{XY})) is true, else $Cx = 1$;
4 setting $Cy = -1$ if ((($\text{abs}(Ry) \geq \text{MAX}_y$) and ($\text{abs}(Ax) > \text{MAX}_X$)) or ($\text{abs}(Ry) \geq$
5 MAX_{XY})) is true, else $Cy = 1$;
6 setting $C'x = 1$ if $R'x$ lies within the boundaries of the second square else $C'x = -1$; and
7 setting $C'y = 1$ if $R'y$ lies within the boundaries of the second square else $C'y = -1$,
8 wherein $R'x$ and $R'y$ are the two-dimensional components of α .

1 17. The method of claim 15, wherein the second computing comprises:
2 computing an intermediate value, d, wherein $d = \text{the received point} - \text{the second point};$
3 setting the third point = the second point + $Cx * \text{sign}(dx) * 2;$
4 setting the fourth point = the second point + $i * Cy * \text{sign}(dy) * 2;$ and
5 setting the fifth point = the second point + $2(Cx * \text{sign}(dx) + i * Cy * \text{sign}(dy)),$
6 wherein Cx and Cy are values specifying if the two-dimensional components of the received
7 point lie inside a boundary of the constellation and dx and dy are two-dimensional components
8 of $d.$

1 18. The method of claim 17, wherein the second computing further comprises:
2 setting a seventh point = the sixth point + $C'x * \text{sign}(d'x) * 2;$ and
3 setting an eighth point = the sixth point + $i * C'y * \text{sign}(d'y) * 2,$
4 wherein $d'x$ and $d'y$ are the two-dimensional values of a second intermediate value, $d',$ wherein
5 $d' = \text{the received point} - \text{the sixth point},$ and $C'x$ and $C'y$ are values specifying if the two
6 dimensional components of the sixth point lie inside a boundary of the second square.

1 19. The method of claim 18, wherein the second computing further comprises checking to
2 ensure that the fifth point is a valid point.

1 20. The method of claim 19, wherein the validity of the fifth point can be checked and
2 corrected using expression:

3 if ($\text{abs}(\text{REAL}(D)) > \text{MAX}_X$) and ($\text{abs}(\text{IMAG}(D)) > \text{MAX}_Y$)
4 { if ($\text{abs}(dx) > \text{abs}(dy)$) then
5 $D = D - 4 * \text{sign}(\text{IMAG}(D)) * i,$
6 else

7 D = D - 4 * sign(REAL(D)) },

8 wherein D is the fifth point, and dx and dy are the two-dimensional components of the fifth
9 point.

1 21. The method of claim 18, wherein the third computing comprises computing a Euclidean
2 distance from the received point to each of the second, third, fourth, fifth, sixth, seventh, and
3 eighth points.

1 22. The method of claim 21, wherein each of the second, third, fourth, fifth, sixth, seventh,
2 and eighth points belongs to one of four unique cosets, and wherein in a coset with more than
3 one point, points with larger Euclidean distances to the received point are discarded.

1 23. The method of claim 1, wherein the method can be used to decode a received point in a
2 communications system.

1 24. The method of claim 23, wherein the communications system is an asymmetric digital
2 subscriber line (ADSL) compliant system.

1 25. A method for computing distances between a received point and four points in a two-
2 dimensional grid with a constellation representing a number of bits equal to three, wherein each
3 of the four points belong to a unique coset in the constellation, the method comprising:
4 computing a point from each coset to the received point; and
5 computing a distance from the received point to the point in each coset.

1 26. The method of claim 25, wherein the first computing comprises:
2 setting a first point = $1 + i$ if $R_x > -1$, else the first point = $-3 + i$;
3 setting a second point = $-1 - i$ if $R_x < 1$, else the second point = $3 - i$;
4 setting a third point = $1 - i$ if $R_y < 1$, else the third point = $1 + 3i$; and
5 setting a fourth point = $-1 + i$ if $R_y > -1$, else the fourth point = $-1 - 3i$;
6 wherein R_x and R_y are the two component values of the received point.

1 27. The method of claim 25, wherein the second computing comprises computing a
2 Euclidean distance between the received point and the point in each coset.